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Research Article

Estimation of Errors in Newton's Divided Difference Formula for Polynomial Interpolation of Functions $x^{1/2}$, $x^{1/3}$ and $x^{1/4}$

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Abstract: Polynomial interpolation of functions $f(x)=x^{1/2}$, $f(x)=x^{1/3}$ and $f(x)=x^{1/4}$ in the interval [1, 2] using Newton's Divided Difference Formula by dividing the interval into 10 equal parts have been done to find the Newton's interpolating polynomials in the forms $f(x) = c_0 + c_1 x + c_2 x^2 + \ldots.. + c_{10} x^{10}$. Actual values, calculated value by Newton's interpolating polynomials, difference between actual and calculated values by Newton's interpolating polynomial, percentage error in the values calculated by Newton's interpolating polynomials at x=1.00, 1.01, 1.02, …, 2.00 have been computed. Average percentage errors in the interpolation of functions $x^{1/2}$, $x^{1/3}$ and $x^{1/4}$ using Newton's divided difference formula show that $x^{1/3}$ is better interpolated as compared to $x^{1/2}$ and $x^{1/4}$.

INTRODUCTION

There are many situations that call for fitting a common function to a collection of data. Determining a function that agrees precisely with the data, at least to within round-off tolerances, is called interpolation [1-2]. In the case of an arbitrary collection of data, polynomial interpolation is the most likely candidate for mainly pragmatic reasons. A polynomial approximation to a collection of data is easy to determine in various ways, and polynomials have easily computed derivatives and integrals that might be useful if the derivative or integral3 [3-5] of the function underlying the data is needed. However, a polynomial of degree n-1 is generally required to satisfy a set of n conditions, and polynomials of even moderately high degree are likely to have a high degree of variation except where explicitly constrained. This means that in order to obtain stable approximating polynomials either the number of specified conditions must be kept small, which may not be a reasonable restriction, or, more likely, the conditions are only required to be satisfied in an approximate way. The problem of approximating [6] a known function with a simpler function follows a pattern similar to that of fitting a function to a collection of data. There are many reasons why such an approximation might be required.

The branch of mathematical statistics devoted to the inference of accurate conclusions about the numerical values of approximately measured quantities, as well as on the errors in the measurements. Repeated measurements of one and the same constant quantity generally give different results, since every measurement contains a certain error. There are three basic types of error: systematic, gross and random. Systematic errors always either overestimate or underestimate the results of measurements and arise for specific reasons (incorrect set-up of measuring equipment, the effect of environment, etc.), which systematically affect the measurements and alter them in one direction. The estimation of systematic errors is achieved using methods which go beyond the confines of mathematical statistics. Gross errors (often called outliers) arise from miscalculations, incorrect reading of the measuring equipment, etc. The results of measurements which contain gross errors differ greatly from other results of measurements and are therefore often easy to identify. Random errors [17-19] arise from various reasons which have an unforeseen effect on each of the measurements, both in overestimating and in underestimating results.

The main contents of approximation theory concern the approximation of functions. Its foundations are laid by the work of P. L. Chebyshev (1 8 5 4 –1 8 5 9) on best uniform approximation of functions by polynomials and by K. Weierstrass, who in 1 8 8 5 established that in principle it is possible to approximate a continuous function on a finite interval by polynomials with arbitrary pre-given error.

Data interpolation[7-10] is one of the most important tasks in geophysical data processing. Its importance is increasing with the development of 3-D seismics, since most of the modern 3-D acquisition geometries carry non-uniform spatial distribution of seismic records. Without a careful interpolation, acquisition irregularities may lead to unwanted artifacts at the imaging step (Gardner and Canning, 1994; Chemingui and Biondi, 1996). The interpolation problem in geophysics implies interpolating irregularly sampled data to a regular grid. In general, this problem requires a regularized inversion scheme, such as the method of inversion to zero offset (Ronen et al., 1991, 1995).

Newton’s forward formula[9] is appropriate to apply only when the unknown value lies near the beginning of the table while Newton’s backward formula is usually applied when the unknown value to be interpolated lies near the end of the table [25]. Lagrange’s formula for interpolation is applicable for any type of observation [4] but when the observations are given at equi-spaced values of arguments, the formulas using various order differences are found to be more convenient and easy to apply rather than Lagrange’s interpolation formula. In the case, the values of the arguments are unequally spaced, we should use Newton’s divided difference formula to represent the function [12-15, 20-24].

Material and Method:

Newton’s divided difference formula: The Lagrange interpolation formula involves very considerable computation and its use can be quite risky. It is much more efficient to use the divided differences method for interpolation [11, 16, 26].

The divided differences for a function f[x] are defined as follows:

$$f[x_{i-1}, x_i] = \frac{f[x_i] - f[x_{i-1}]}{x_i - x_{i-1}}$$

$$f[x_{i-2}, x_{i-1}, x_i] = \frac{f[x_{i-1}, x_i] - f[x_{i-2}, x_{i-1}]}{x_i - x_{i-2}}$$

$$f[x_{i-3}, x_{i-2}, x_{i-1}, x_i] = \frac{f[x_{i-2}, x_{i-1}, x_i] - f[x_{i-3}, x_{i-2}, x_{i-1}]}{x_i - x_{i-3}}$$

$$f[x_{i-j}, x_{i-j+1}, \ldots, x_i] = \frac{f[x_{i-j+1}, \ldots, x_i] - f[x_{i-j}, \ldots, x_{i-1}]}{x_i - x_{i-j}}$$

The coefficient a_i of the Newton polynomial $P_n(x)$ is

$a_i=f_n[x_0, x_1, x_2, \ldots, x_n]$ and it is the top element in the column of the i-th divided differences.

The Newton polynomial of degree $\leq$ n that passes through the n+1 points

$(x_k, y_k) = (x_k, f(x_k))$, for k=0, 1, …, n is

$$P_n(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + a_3(x-x_0)(x-x_1)(x-x_2) + \ldots + a_n(x-x_0)(x-x_1)(x-x_2)\ldots(x-x_{n-1})$$

We have divided the interval [a, b] into 10 equal parts with the help of the points

$a=x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}=b$

and calculated $y_i = f(x_i)$ where i=0,1, 2, …. , 10. We have also drawn difference triangle for the function y=f(x). Difference triangle has been drawn by using the formula

$d^n y_i = d^{n-1} y_i - d^{n-1} y_{i-1}$ where n, i = 0,1, 2, …. , 10

Now, the Newton's divided difference formula

$f(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + \ldots. + a_{10}(x-x_0)(x-x_1)\ldots.(x-x_9)$

where $a_n=(d^n y_0)/(n!h^n)$; n = 0,1, 2, …. , 10

becomes

$f(x) = c_0 + c_1 x + c_2 x^2 + \ldots.. + c_{10} x^{10}$

where

c0=a0-a1*p(1,1)+a2*p(2,2)-a3*p(3,3)+a4*p(4,4)-a5*p(5,5)+a6*p(6,6)-a7*p(7,7)
+a8*p(8,8) -a9*p(9,9)+a10*p(10,10);

c1=a1-a2*p(2,1)+a3*p(3,2)-a4*p(4,3)+a5*p(5,4)-a6*p(6,5)+a7*p(7,6)-a8*p(8,7)
+a9*p(9,8) -a10*p(10,9);

c2=a2-a3*p(3,1)+a4*p(4,2)-a5*p(5,3)+a6*p(6,4)-a7*p(7,5)+a8*p(8,6)-a9*p(9,7)
+a10*p(10,8);

c3=a3-a4*p(4,1)+a5*p(5,2)-a6*p(6,3)+a7*p(7,4)-a8*p(8,5)+a9*p(9,6)-a10*p(10,7);

c4=a4-a5*p(5,1)+a6*p(6,2)-a7*p(7,3)+a8*p(8,4)-a9*p(9,5)+a10*p(10,6);

c5=a5-a6*p(6,1)+a7*p(7,2)-a8*p(8,3)+a9*p(9,4)-a10*p(10,5);

c6=a6-a7*p(7,1)+a8*p(8,2)-a9*p(9,3)+a10*p(10,4);

c7=a7-a8*p(8,1)+a9*p(9,2)-a10*p(10,3);

c8=a8-a9*p(9,1)+a10*p(10,2);

c9=a9-a10*p(10,1);

c10=a10;

p(i,j)=summation of the product of j elements in all combinations among $x_0,x_1,..,x_{i-1}$

For obtaining the polynomial interpolation of the functions $x^{1/2}$, $x^{1/3}$ and $x^{1/4}$ using Newton's divided difference formula in the interval [1, 2], a Program in C++ computer programming language has been developed.

RESULT AND DISCUSSION

Interpolation of $f(x)=x^{1/2}$ in the interval [1, 2]: We have divided the interval into 10 equal parts with the help of the points x_0, x_1, ….., x_{10} such that

$x_i = x_0+ih$, i=0,1,2,…., 10, h=(b-a)/n, n=10

Polynomial obtained by Newton's divided difference formula using the developed C++ program is given below-

$f(x) = c_0 + c_1 x + c_2 x^2 + ….. + c_{10} x^{10}$

where

$c_0 = -1.944630265236$

$c_1 = 16.710096359253$

$c_2 = -49.732860565186$

$c_3 = 91.514305114746$

$c_4 = -110.665145874023$

$c_5 = 91.462081909180$

$c_6 = -52.255199432373$

$c_7 = 20.369894027710$

$c_8 = -5.184066295624$

$c_9 = 0.777744591236$

$c_{10} = -0.052232898772$

Difference triangle for the function $f(x)=x^{1/2}$ at the points of interval [1, 2] is given in Table-1. Actual value of $f(x)=x^{1/2}$, Calculated value of $f(x)=x^{1/2}$ by Newton's interpolating polynomial, Difference between actual and calculated values of $f(x)=x^{1/2}$ by Newton's interpolating polynomial, Percentage error in the values of $f(x)=x^{1/2}$ calculated by Newton's interpolating polynomial at different values of x is given in Table-2. Graph between actual values of $f(x)=x^{1/2}$ and the values calculated by Newton's interpolating polynomial at different points in the interval [1, 2] separated by the distance 0.1 is given in Graph-1. Difference between actual and calculated values of the function $f(x)=x^{1/2}$ by Newton's interpolating polynomial is shown in Graph-2. Average percentage error in the values obtained by Newton's interpolating polynomial is 0.00234432%.

Graph 1: Graph between actual values of $f(x) = x^{1/2}$ and the values calculated by Newton's interpolating polynomial at different points in the interval [1, 2] separated by the distance 0.1

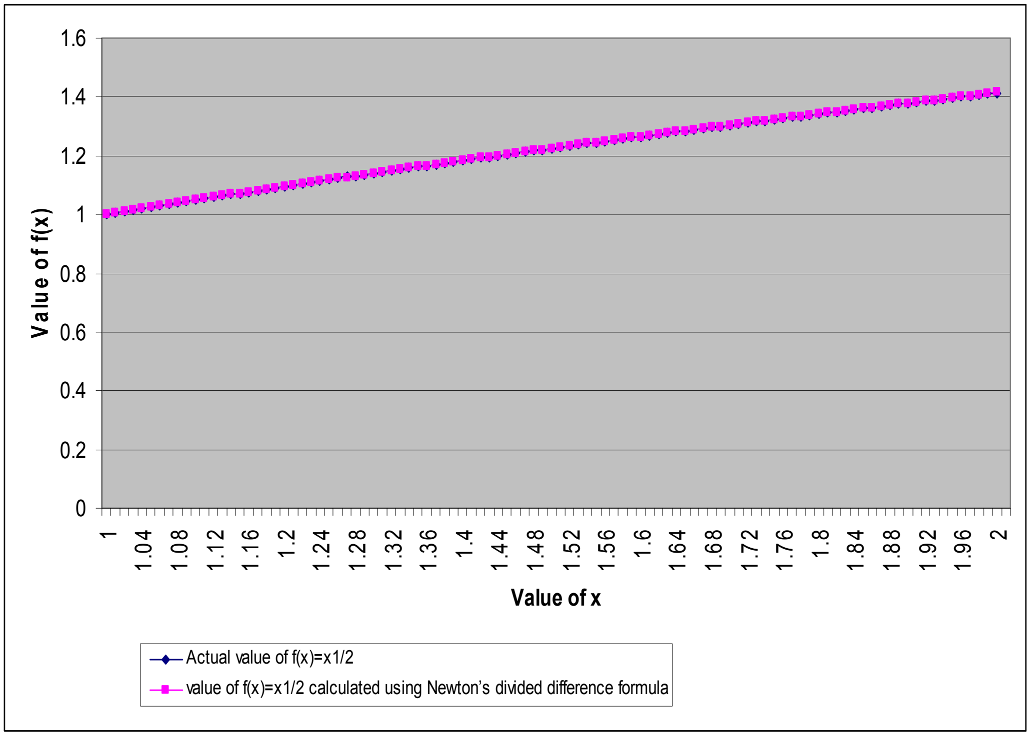


Table-1: Difference triangle for the function $f(x)=x^{1/2}$ at points of interval [1, 2]

x	y=f(x)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$	$\Delta^7 y$	$\Delta^8 y$	$\Delta^9 y$	$\Delta^{10} y$
1.00000000	1.00000000										
		0.04880881									
1.10000002	1.04880881		-0.00217247								
		0.04663634		0.00026643							
1.20000005	1.09544516		-0.00190604		-0.00005019						
		0.04473031		0.00021625		0.00001216					
1.29999995	1.14017546		-0.00168979		-0.00003803		-0.00000358				
		0.04304051		0.00017822		0.00000858		0.00000179			
1.39999998	1.18321598		-0.00151157		-0.00002944		-0.00000179		-0.00000286		
		0.04152894		0.00014877		0.00000679		-0.00000107		0.00000739	
1.50000000	1.22474492		-0.00136280		-0.00002265		-0.00000286		0.00000453		-0.00001895
		0.04016614		0.00012612		0.00000393		0.00000346		-0.00001156	
1.60000002	1.26491106		-0.00123668		-0.00001872		0.00000060		-0.00000703		
		0.03892946		0.00010741		0.00000453		-0.00000358			
1.70000005	1.30384052		-0.00112927		-0.00001419		-0.00000298				
		0.03780019		0.00009322		0.00000155					
1.79999995	1.34164071		-0.00103605		-0.00001264						
		0.03676414		0.00008059							
1.89999998	1.37840486		-0.00095546								
		0.03580868									
2.00000000	1.41421354										

Table-2: Actual value of $f(x)=x^{1/2}$, Calculated value of $f(x)=x^{1/2}$ by Newton's interpolating polynomial, Difference between actual and calculated values of $f(x)=x^{1/2}$ by Newton's interpolating polynomial, Percentage error in the values of $f(x)=x^{1/2}$ calculated by Newton's interpolating polynomial at different values of x

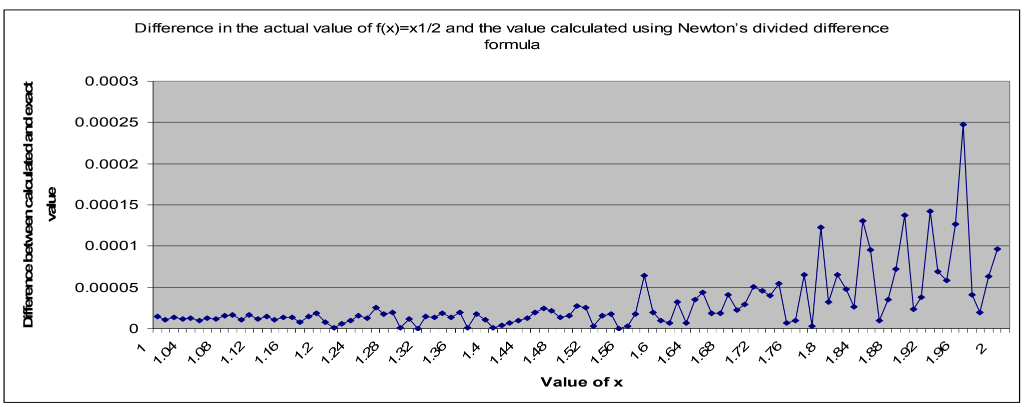
Value of x	Actual value of $f(x)=x^{1/2}$	Calculated value of $f(x)=x^{1/2}$	Difference between the actual value of $f(x)=x^{1/2}$ and calculated value	Percentage error in the calculated value
1.00	1.00000000	0.99998528	0.00001472	0.00147223
1.01	1.00498760	1.00497699	0.00001061	0.00105570
1.02	1.00995052	1.00993705	0.00001347	0.00133379
1.03	1.01488912	1.01487720	0.00001192	0.00117460
1.04	1.01980388	1.01979160	0.00001228	0.00120401
1.05	1.02469504	1.02468491	0.00001013	0.00098886
1.06	1.02956295	1.02954996	0.00001299	0.00126207
1.07	1.03440797	1.03439629	0.00001168	0.00112939
1.08	1.03923047	1.03921461	0.00001585	0.00152563
1.09	1.04403067	1.04401422	0.00001645	0.00157571
1.10	1.04880881	1.04879856	0.00001025	0.00097749
1.11	1.05356538	1.05354846	0.00001693	0.00160671
1.12	1.05830050	1.05828881	0.00001168	0.00110389
1.13	1.06301451	1.06299996	0.00001454	0.00136814
1.14	1.06770778	1.06769753	0.00001025	0.00096019
1.15	1.07238042	1.07236683	0.00001359	0.00126726
1.16	1.07703292	1.07701898	0.00001395	0.00129499
1.17	1.08166528	1.08165741	0.00000787	0.00072738
1.18	1.08627796	1.08626378	0.00001419	0.00130592
1.19	1.09087110	1.09085298	0.00001812	0.00166104
1.20	1.09544504	1.09543693	0.00000811	0.00073999
1.21	1.09999990	1.09999859	0.00000131	0.00011921
1.22	1.10453606	1.10453069	0.00000536	0.00048567
1.23	1.10905349	1.10904360	0.00000989	0.00089215
1.24	1.11355281	1.11353695	0.00001585	0.00142381
1.25	1.11803389	1.11802137	0.00001252	0.00111955
1.26	1.12249708	1.12247193	0.00002515	0.00224082

Value of x	Actual value of $f(x)=x^{1/2}$	Calculated value of $f(x)=x^{1/2}$	Difference between the actual value of $f(x)=x^{1/2}$ and calculated value	Percentage error in the calculated value
1.27	1.12694263	1.12692499	0.00001764	0.00156556
1.28	1.13137078	1.13135087	0.00001991	0.00175963
1.29	1.13578153	1.13578272	0.00000119	0.00010496
1.30	1.14017534	1.14016378	0.00001156	0.00101417
1.31	1.14455223	1.14455187	0.00000036	0.00003125
1.32	1.14891243	1.14889753	0.00001490	0.00129698
1.33	1.15325618	1.15324283	0.00001335	0.00115772
1.34	1.15758359	1.15756488	0.00001872	0.00161680
1.35	1.16189492	1.16188109	0.00001383	0.00119015
1.36	1.16619027	1.16617095	0.00001931	0.00165598
1.37	1.17046988	1.17047131	0.00000143	0.00012222
1.38	1.17473388	1.17471647	0.00001740	0.00148157
1.39	1.17898250	1.17897201	0.00001049	0.00088979
1.40	1.18321574	1.18321478	0.00000095	0.00008060
1.41	1.18743408	1.18743837	0.00000429	0.00036141
1.42	1.19163740	1.19163036	0.00000703	0.00059023
1.43	1.19582593	1.19583559	0.00000966	0.00080747
1.44	1.19999981	1.20001245	0.00001264	0.00105302
1.45	1.20415926	1.20417917	0.00001991	0.00165327
1.46	1.20830441	1.20832837	0.00002396	0.00198303
1.47	1.21243536	1.21241379	0.00002158	0.00177963
1.48	1.21655238	1.21656597	0.00001359	0.00111708
1.49	1.22065532	1.22067058	0.00001526	0.00125005
1.50	1.22474468	1.22477233	0.00002766	0.00225815
1.51	1.22882032	1.22884560	0.00002527	0.00205664
1.52	1.23288262	1.23288012	0.00000250	0.00020305
1.53	1.23693144	1.23691595	0.00001550	0.00125288
1.54	1.24096715	1.24094939	0.00001776	0.00143132
1.55	1.24498975	1.24498940	0.00000036	0.00002873
1.56	1.24899936	1.24899673	0.00000262	0.00020998
1.57	1.25299621	1.25301385	0.00001764	0.00140806

Value of x	Actual value of $f(x)=x^{1/2}$	Calculated value of $f(x)=x^{1/2}$	Difference between the actual value of $f(x)=x^{1/2}$ and calculated value	Percentage error in the calculated value
1.58	1.25698030	1.25704503	0.00006473	0.00514969
1.59	1.26095176	1.26097143	0.00001967	0.00155990
1.60	1.26491082	1.26492095	0.00001013	0.00080107
1.61	1.26885748	1.26886439	0.00000691	0.00054491
1.62	1.27279198	1.27282429	0.00003231	0.00253818
1.63	1.27671432	1.27672136	0.00000703	0.00055089
1.64	1.28062463	1.28065991	0.00003529	0.00275537
1.65	1.28452301	1.28456652	0.00004351	0.00338736
1.66	1.28840959	1.28842819	0.00001860	0.00144338
1.67	1.29228461	1.29230285	0.00001824	0.00141138
1.68	1.29614794	1.29618931	0.00004137	0.00319143
1.69	1.29999971	1.30002189	0.00002217	0.00170561
1.70	1.30384028	1.30386996	0.00002968	0.00227659
1.71	1.30766940	1.30771983	0.00005043	0.00385614
1.72	1.31148744	1.31153333	0.00004590	0.00349951
1.73	1.31529438	1.31533432	0.00003994	0.00303621
1.74	1.31909037	1.31914496	0.00005460	0.00413905
1.75	1.32287538	1.32286859	0.00000679	0.00051365
1.76	1.32664967	1.32664025	0.00000942	0.00070987
1.77	1.33041322	1.33047867	0.00006545	0.00491922
1.78	1.33416617	1.33416867	0.00000250	0.00018764
1.79	1.33790851	1.33803117	0.00012267	0.00916852
1.80	1.34164047	1.34167218	0.00003171	0.00236350
1.81	1.34536207	1.34542727	0.00006521	0.00484684
1.82	1.34907341	1.34912097	0.00004756	0.00352572
1.83	1.35277462	1.35274851	0.00002611	0.00192987
1.84	1.35646570	1.35659647	0.00013077	0.00964069
1.85	1.36014676	1.36024237	0.00009561	0.00702908
1.86	1.36381781	1.36380804	0.00000978	0.00071675
1.87	1.36747909	1.36751378	0.00003469	0.00253678
1.88	1.37113059	1.37120223	0.00007164	0.00522523

Value of x	Actual value of $f(x)=x^{1/2}$	Calculated value of $f(x)=x^{1/2}$	Difference between the actual value of $f(x)=x^{1/2}$ and calculated value	Percentage error in the calculated value
1.89	1.37477243	1.37491024	0.00013781	0.01002391
1.90	1.37840462	1.37838101	0.00002360	0.00171237
1.91	1.38202715	1.38198936	0.00003779	0.00273434
1.92	1.38564038	1.38578224	0.00014186	0.01023780
1.93	1.38924408	1.38917506	0.00006902	0.00496833
1.94	1.39283848	1.39289713	0.00005865	0.00421090
1.95	1.39642370	1.39654994	0.00012624	0.00904042
1.96	1.39999962	1.40024686	0.00024724	0.01766001
1.97	1.40356660	1.40352559	0.00004101	0.00292170
1.98	1.40712440	1.40714371	0.00001931	0.00137244
1.99	1.41067326	1.41073632	0.00006306	0.00447033
2.00	1.41421318	1.41430986	0.00009668	0.00683622
Average percentage error				**0.00234432**

Graph 2: Difference between actual and calculated values of the function $f(x) = x^{1/2}$ by Newton's interpolating polynomial



Interpolation of $f(x)=x^{1/3}$ in the interval [1, 2]: We have divided the interval into 10 equal parts with the help of the points $x_0, x_1, \ldots, x_{10}$ such that

$x_i=x_0+ih$, $i=0,1,2,\ldots, 10$, $h=(b-a)/n$, $n=10$

Polynomial obtained by Newton's divided difference formula using the developed C++ program is given below-

$f(x) = c_0 + c_1 x + c_2 x^2 + \ldots.. + c_{10} x^{10}$

where

c0= -1.038121581078

c1= 11.380318641663

c2= -33.470668792725

c3= 61.235923767090

c4= -73.864898681641

c5= 60.978164672852

c6= -34.822807312012

c7= 13.573181152344

c8= -3.454664230347

c9= 0.518386840820

c10= -0.034821931273

Difference triangle for the function $f(x)=x^{1/3}$ at the points of interval [1, 2] is given in Table-3. Actual value of $f(x)=x^{1/3}$, Calculated value of $f(x)=x^{1/3}$ by Newton's interpolating polynomial, Difference between actual and calculated values of $f(x)=x^{1/3}$ by Newton's interpolating polynomial, Percentage error in the values of $f(x)=x^{1/3}$ calculated by Newton's interpolating polynomial at different values of x is given in Table-4. Graph between actual values of $f(x)=x^{1/3}$ and the values calculated by Newton's interpolating polynomial at different points in the interval [1, 2] separated by the distance 0.01 is given in Graph-3. Difference between actual and calculated values of the function $f(x)=x^{1/3}$ by Newton's interpolating polynomial is shown in Graph-4. Average percentage error in the values obtained by Newton's interpolating polynomial is 0.00222263%.

Graph 3: Graph between actual values of $f(x) = x^{1/3}$ and the values calculated by Newton's interpolating polynomial at different points in the interval [1, 2] separated by the distance 0.1

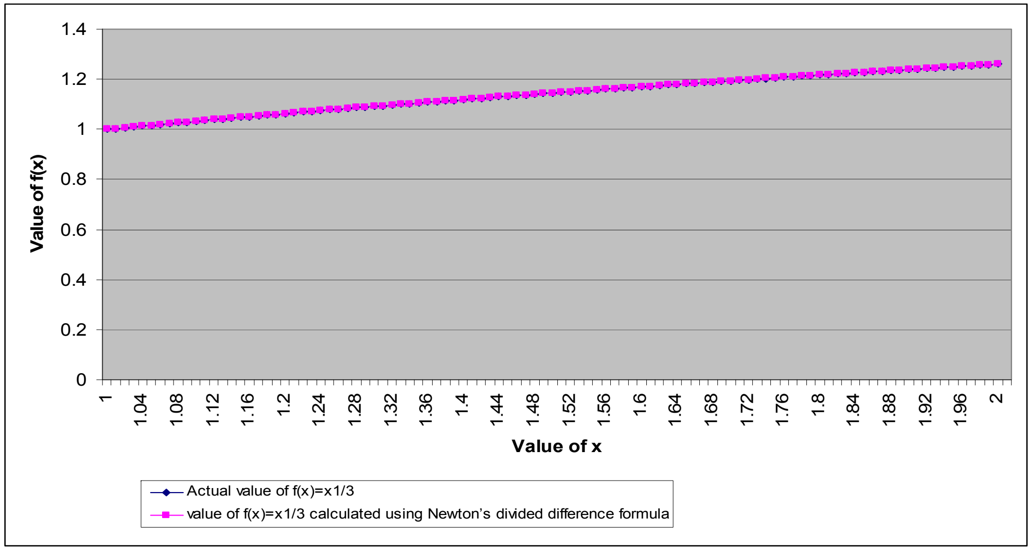

1.4
1.2
1
0.8
0.6
0.4
0.2
0
Value of f(x)
1
1.04
1.08
1.12
1.16
1.2
1.24
1.28
1.32
1.36
1.4
1.44
1.48
1.52
1.56
1.6
1.64
1.68
1.72
1.76
1.8
1.84
1.88
1.92
1.96
2
Value of x
Actual value of f(x)=x1/3
value of f(x)=x1/3 calculated using Newton's divided difference formula

Table 3: Difference triangle for the function $f(x)=x^{1/3}$ at points of interval [1, 2]

x	y=f(x)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$	$\Delta^7 y$	$\Delta^8 y$	$\Delta^9 y$	$\Delta^{10} y$
1.00000000	1.00000000										
		0.03228009									
1.10000002	1.03228009		-0.00190163								
		0.03037846		0.00025749							
1.20000005	1.06265855		-0.00164413		-0.00005162						
		0.02873433		0.00020587		0.00001323					
1.29999995	1.09139287		-0.00143826		-0.00003839		-0.00000405				
		0.02729607		0.00016749		0.00000918		0.00000167			
1.39999998	1.11868894		-0.00127077		-0.00002921		-0.00000238		-0.00000191		
		0.02602530		0.00013828		0.00000679		-0.00000024		0.00000489	
1.50000000	1.14471424		-0.00113249		-0.00002241		-0.00000262		0.00000298		-0.00001264
		0.02489281		0.00011587		0.00000417		0.00000274		-0.00000775	
1.60000002	1.16960704		-0.00101662		-0.00001824		0.00000012		-0.00000477		
		0.02387619		0.00009763		0.00000429		-0.00000203			
1.70000005	1.19348323		-0.00091898		-0.00001395		-0.00000191				
		0.02295721		0.00008368		0.00000238					
1.79999995	1.21644044		-0.00083530		-0.00001156						
		0.02212191		0.00007212							
1.89999998	1.23856235		-0.00076318								
		0.02135873									
2.00000000	1.25992107										

Graph-4: Difference between actual and calculated values of the function $f(x) = x^{1/3}$ by Newton's interpolating polynomial

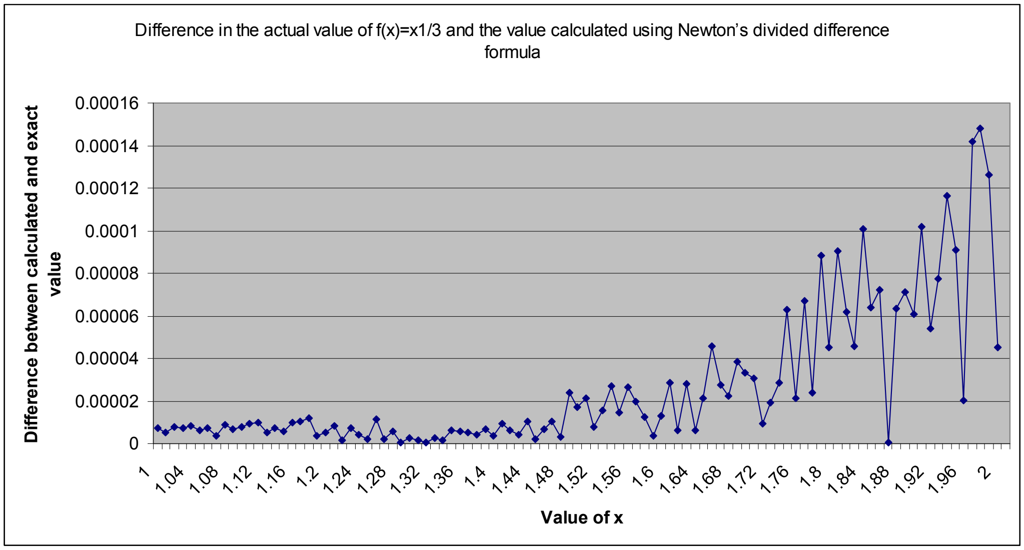


Table 4: Actual value of $f(x)=x^{1/3}$, Calculated value of $f(x)=x^{1/3}$ by Newton's interpolating polynomial, Difference between actual and calculated values of $f(x)=x^{1/3}$ by Newton's interpolating polynomial, Percentage error in the values of $f(x)=x^{1/3}$ calculated by Newton's interpolating polynomial at different values of x

Value of x	Actual value of $f(x)=x^{1/3}$	Calculated value of $f(x)=x^{1/3}$	Difference between the actual value of $f(x)=x^{1/3}$ and calculated value	Percentage error in the calculated value
1.00	1.00000000	0.99999291	0.00000709	0.00070930
1.01	1.00332224	1.00331688	0.00000536	0.00053467
1.02	1.00662267	1.00661480	0.00000787	0.00078160
1.03	1.00990164	1.00989437	0.00000727	0.00072005
1.04	1.01315939	1.01315105	0.00000834	0.00082363
1.05	1.01639628	1.01638997	0.00000632	0.00062162
1.06	1.01961279	1.01960528	0.00000751	0.00073657
1.07	1.02280915	1.02280533	0.00000381	0.00037296
1.08	1.02598560	1.02597702	0.00000858	0.00083657
1.09	1.02914250	1.02913582	0.00000668	0.00064867
1.10	1.03228009	1.03227246	0.00000763	0.00073908
1.11	1.03539872	1.03538930	0.00000942	0.00090956
1.12	1.03849876	1.03848886	0.00000989	0.00095276
1.13	1.04158044	1.04157543	0.00000501	0.00048069

Value of x	Actual value of $f(x)=x^{1/3}$	Calculated value of $f(x)=x^{1/3}$	Difference between the actual value of $f(x)=x^{1/3}$ and calculated value	Percentage error in the calculated value
1.14	1.04464388	1.04463685	0.00000703	0.00067328
1.15	1.04768956	1.04768360	0.00000596	0.00056892
1.16	1.05071747	1.05070758	0.00000989	0.00094168
1.17	1.05372822	1.05371797	0.00001025	0.00097293
1.18	1.05672181	1.05670989	0.00001192	0.00112810
1.19	1.05969846	1.05969501	0.00000346	0.00032623
1.20	1.06265855	1.06265330	0.00000525	0.00049359
1.21	1.06560218	1.06559408	0.00000811	0.00076072
1.22	1.06852973	1.06852794	0.00000179	0.00016735
1.23	1.07144117	1.07143366	0.00000751	0.00070094
1.24	1.07433701	1.07434130	0.00000429	0.00039946
1.25	1.07721722	1.07721949	0.00000226	0.00021026
1.26	1.08008218	1.08007097	0.00001121	0.00103748
1.27	1.08293211	1.08292997	0.00000215	0.00019814
1.28	1.08576703	1.08576155	0.00000548	0.00050505
1.29	1.08858716	1.08858776	0.00000060	0.00005475
1.30	1.09139276	1.09139001	0.00000274	0.00025122
1.31	1.09418404	1.09418249	0.00000155	0.00014163
1.32	1.09696126	1.09696090	0.00000036	0.00003260
1.33	1.09972441	1.09972692	0.00000250	0.00022764
1.34	1.10247374	1.10247219	0.00000155	0.00014057
1.35	1.10520935	1.10521555	0.00000620	0.00056088
1.36	1.10793161	1.10793734	0.00000572	0.00051646
1.37	1.11064041	1.11064553	0.00000513	0.00046154
1.38	1.11333621	1.11334050	0.00000429	0.00038547
1.39	1.11601889	1.11602581	0.00000691	0.00061954
1.40	1.11868882	1.11869228	0.00000346	0.00030903
1.41	1.12134612	1.12135541	0.00000930	0.00082921
1.42	1.12399077	1.12399697	0.00000620	0.00055151
1.43	1.12662303	1.12662745	0.00000441	0.00039150
1.44	1.12924314	1.12925375	0.00001061	0.00093953
1.45	1.13185108	1.13185334	0.00000226	0.00020011

Value of x	Actual value of $f(x)=x^{1/3}$	Calculated value of $f(x)=x^{1/3}$	Difference between the actual value of $f(x)=x^{1/3}$ and calculated value	Percentage error in the calculated value
1.46	1.13444710	1.13444054	0.00000656	0.00057795
1.47	1.13703120	1.13704145	0.00001025	0.00090165
1.48	1.13960373	1.13960695	0.00000322	0.00028244
1.49	1.14216459	1.14218831	0.00002372	0.00207699
1.50	1.14471412	1.14473140	0.00001729	0.00151001
1.51	1.14725232	1.14727378	0.00002146	0.00187035
1.52	1.14977932	1.14978719	0.00000787	0.00068429
1.53	1.15229523	1.15231061	0.00001538	0.00133455
1.54	1.15480018	1.15482724	0.00002706	0.00234331
1.55	1.15729439	1.15730870	0.00001431	0.00123608
1.56	1.15977788	1.15980458	0.00002670	0.00230241
1.57	1.16225076	1.16227067	0.00001991	0.00171288
1.58	1.16471314	1.16472554	0.00001240	0.00106445
1.59	1.16716516	1.16716886	0.00000370	0.00031662
1.60	1.16960692	1.16962004	0.00001311	0.00112115
1.61	1.17203856	1.17206728	0.00002873	0.00245124
1.62	1.17446017	1.17446625	0.00000608	0.00051766
1.63	1.17687178	1.17690003	0.00002825	0.00240065
1.64	1.17927361	1.17927992	0.00000632	0.00053576
1.65	1.18166566	1.18168700	0.00002134	0.00180580
1.66	1.18404806	1.18409383	0.00004578	0.00386609
1.67	1.18642080	1.18644810	0.00002730	0.00230095
1.68	1.18878424	1.18880665	0.00002241	0.00188523
1.69	1.19113827	1.19117665	0.00003839	0.00322258
1.70	1.19348300	1.19351614	0.00003314	0.00277676
1.71	1.19581866	1.19584906	0.00003040	0.00254206
1.72	1.19814515	1.19815469	0.00000954	0.00079596
1.73	1.20046258	1.20048201	0.00001943	0.00161864
1.74	1.20277119	1.20279968	0.00002849	0.00236878
1.75	1.20507097	1.20513391	0.00006294	0.00522314
1.76	1.20736194	1.20738328	0.00002134	0.00176736
1.77	1.20964432	1.20971143	0.00006711	0.00554831

Value of x	Actual value of $f(x)=x^{1/3}$	Calculated value of $f(x)=x^{1/3}$	Difference between the actual value of $f(x)=x^{1/3}$ and calculated value	Percentage error in the calculated value
1.78	1.21191812	1.21194196	0.00002384	0.00196728
1.79	1.21418333	1.21427155	0.00008821	0.00726537
1.80	1.21644020	1.21648562	0.00004542	0.00373374
1.81	1.21868873	1.21877897	0.00009024	0.00740480
1.82	1.22092903	1.22099090	0.00006187	0.00506742
1.83	1.22316098	1.22320652	0.00004554	0.00372297
1.84	1.22538495	1.22548568	0.00010073	0.00822042
1.85	1.22760081	1.22766459	0.00006378	0.00519525
1.86	1.22980881	1.22988081	0.00007200	0.00585476
1.87	1.23200881	1.23200941	0.00000060	0.00004838
1.88	1.23420095	1.23426425	0.00006330	0.00512884
1.89	1.23638535	1.23645651	0.00007117	0.00575613
1.90	1.23856211	1.23862290	0.00006080	0.00490865
1.91	1.24073124	1.24083281	0.00010157	0.00818600
1.92	1.24289286	1.24294686	0.00005400	0.00434485
1.93	1.24504685	1.24512422	0.00007737	0.00621397
1.94	1.24719346	1.24731004	0.00011659	0.00934792
1.95	1.24933279	1.24942350	0.00009072	0.00726134
1.96	1.25146472	1.25148511	0.00002038	0.00162887
1.97	1.25358951	1.25373137	0.00014186	0.01131623
1.98	1.25570703	1.25585485	0.00014782	0.01177182
1.99	1.25781751	1.25794351	0.00012600	0.01001769
2.00	1.25992084	1.25996614	0.00004530	0.00359543
Average percentage error				**0.00222263**

Interpolation of $f(x)=x^{1/4}$ in the interval [1, 2]: We have divided the interval into 10 equal parts with the help of the points $x_0, x_1, \ldots, x_{10}$ such that

$x_i=x_0+ih,\ i=0,1,2,\ldots,10,\ h=(b-a)/n,\ n=10$

Polynomial obtained by Newton's divided difference formula using the developed C++ program is given below-

$f(x) = c_0 + c_1 x + c_2 x^2 + \ldots + c_{10} x^{10}$

where

$c_0=$ 2.534127712250

$c_1=$ -12.973608970642

$c_2=$ 42.297733306885

$c_3=$ -78.338096618652

$c_4=$ 94.002273559570

$c_5=$ -76.714340209961

$c_6=$ 43.183174133301

$c_7=$ -16.567003250122

$c_8=$ 4.147193908691

$c_9=$ -0.611847639084

$c_{10}=$ 0.040406581014

Difference triangle for the function $f(x)=x^{1/4}$ at the points of interval [1, 2] is given in Table-5. Actual value of $f(x)=x^{1/4}$, Calculated value of $f(x)=x^{1/4}$ by Newton's interpolating polynomial, Difference between actual and calculated values of $f(x)=x^{1/4}$ by Newton's interpolating polynomial, Percentage error in the values of $f(x)=x^{1/4}$ calculated by Newton's interpolating polynomial at different values of x is given in Table-6. Graph between actual values of $f(x)=x^{1/4}$ and the values calculated by Newton's interpolating polynomial at different points in the interval [1, 2] separated by the distance 0.01 is given in Graph-5. Difference between actual and calculated values of the function $f(x)=x^{1/4}$ by Newton's interpolating polynomial is shown in Graph-6. Average percentage error in the values obtained by Newton's interpolating polynomial is 0.00240526 %.

Graph 5: Graph between actual values of $f(x) = x^{1/4}$ and the values calculated by Newton's interpolating polynomial at different points in the interval [1, 2] separated by the distance 0.1

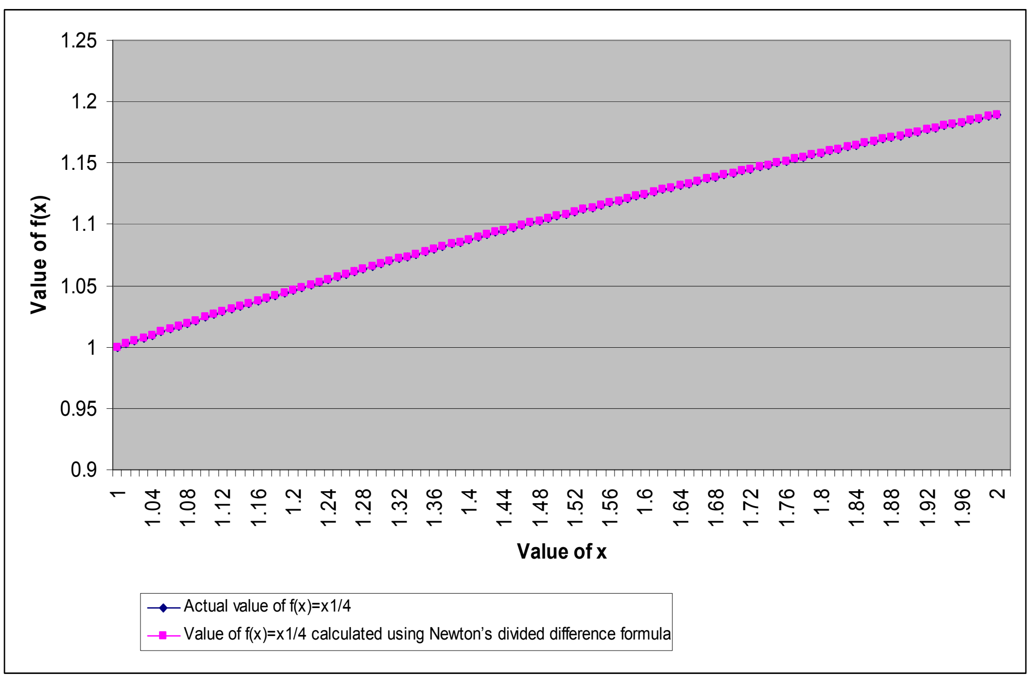


Table 5: Difference triangle for the function $f(x)=x^{1/2}$ at points of interval [1, 2]

x	y=f(x)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$	$\Delta^7 y$	$\Delta^8 y$	$\Delta^9 y$	$\Delta^{10} y$
1.00000000	1.00000000										
		0.02411366									
1.10000002	1.02411366		-0.00159216								
		0.02252150		0.00022542							
1.20000005	1.04663515		-0.00136673		-0.00004601						
		0.02115476		0.00017941		0.00001109					
1.29999995	1.06778991		-0.00118732		-0.00003493		-0.00000167				
		0.01996744		0.00014448		0.00000942		-0.00000238			
1.39999998	1.08775735		-0.00104284		-0.00002551		-0.00000405		0.00000560		
		0.01892459		0.00011897		0.00000536		0.00000322		-0.00000942	
1.50000000	1.10668194		-0.00092387		-0.00002015		-0.00000083		-0.00000381		0.00001466
		0.01800072		0.00009882		0.00000453		-0.00000060		0.00000525	
1.60000002	1.12468266		-0.00082505		-0.00001562		-0.00000143		0.00000143		
		0.01717567		0.00008321		0.00000310		0.00000083			
1.70000005	1.14185834		-0.00074184		-0.00001252		-0.00000060				
		0.01643384		0.00007069		0.00000250					
1.79999995	1.15829217		-0.00067115		-0.00001001						
		0.01576269		0.00006068							
1.89999998	1.17405486		-0.00061047								
		0.01515222									
2.00000000	1.18920708										

Graph 6: Difference between actual and calculated values of the function $f(x) = x^{1/4}$ by Newton's interpolating polynomial

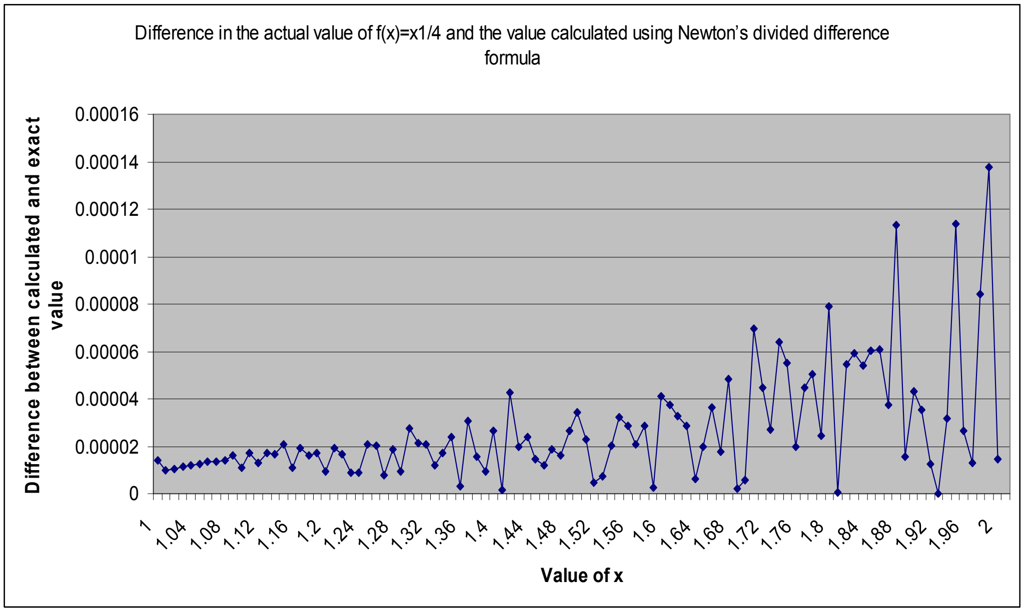


Table 6: Actual value of $f(x)=x^{1/4}$, Calculated value of $f(x)=x^{1/4}$ by Newton's interpolating polynomial, Difference between actual and calculated values of $f(x)=x^{1/4}$ by Newton's interpolating polynomial, Percentage error in the values of $f(x)=x^{1/4}$ calculated by Newton's interpolating polynomial at different values of x

Value of x	Actual value of $f(x)=x^{1/4}$	Calculated value of $f(x)=x^{1/4}$	Difference between the actual value of $f(x)=x^{1/4}$ and calculated value	Percentage error in the calculated value
1.00	1.00000000	1.00001395	0.00001395	0.00139475
1.01	1.00249064	1.00250065	0.00001001	0.00099887
1.02	1.00496292	1.00497353	0.00001061	0.00105572
1.03	1.00741708	1.00742853	0.00001144	0.00113598
1.04	1.00985336	1.00986528	0.00001192	0.00118046
1.05	1.01227224	1.01228452	0.00001228	0.00121297
1.06	1.01467383	1.01468742	0.00001359	0.00133933
1.07	1.01705849	1.01707208	0.00001359	0.00133619
1.08	1.01942658	1.01944077	0.00001419	0.00139156
1.09	1.02177811	1.02179408	0.00001597	0.00156336
1.10	1.02411366	1.02412462	0.00001097	0.00107090

Value of x	Actual value of $f(x)=x^{1/4}$	Calculated value of $f(x)=x^{1/4}$	Difference between the actual value of $f(x)=x^{1/4}$ and calculated value	Percentage error in the calculated value
1.11	1.02643335	1.02645040	0.00001705	0.00166079
1.12	1.02873731	1.02875054	0.00001323	0.00128626
1.13	1.03102601	1.03104305	0.00001705	0.00165339
1.14	1.03329945	1.03331625	0.00001681	0.00162668
1.15	1.03555799	1.03557885	0.00002086	0.00201453
1.16	1.03780198	1.03781271	0.00001073	0.00103380
1.17	1.04003143	1.04005075	0.00001931	0.00185686
1.18	1.04224658	1.04226267	0.00001609	0.00154409
1.19	1.04444778	1.04446483	0.00001705	0.00163215
1.20	1.04663515	1.04664433	0.00000918	0.00087701
1.21	1.04880881	1.04882824	0.00001943	0.00185268
1.22	1.05096912	1.05098581	0.00001669	0.00158799
1.23	1.05311608	1.05312502	0.00000894	0.00084898
1.24	1.05525005	1.05525887	0.00000882	0.00083596
1.25	1.05737126	1.05739188	0.00002062	0.00195042
1.26	1.05947959	1.05949986	0.00002027	0.00191279
1.27	1.06157553	1.06158316	0.00000763	0.00071869
1.28	1.06365907	1.06367779	0.00001872	0.00175957
1.29	1.06573057	1.06573987	0.00000930	0.00087248
1.30	1.06778991	1.06781721	0.00002730	0.00255658
1.31	1.06983745	1.06985855	0.00002110	0.00197227
1.32	1.07187331	1.07189405	0.00002074	0.00193516
1.33	1.07389760	1.07390952	0.00001192	0.00111006
1.34	1.07591057	1.07592750	0.00001693	0.00157334
1.35	1.07791221	1.07793605	0.00002384	0.00221186
1.36	1.07990289	1.07990575	0.00000286	0.00026493
1.37	1.08188260	1.08191311	0.00003052	0.00282078
1.38	1.08385146	1.08386707	0.00001562	0.00144083
1.39	1.08580959	1.08581901	0.00000942	0.00086733
1.40	1.08775723	1.08778381	0.00002658	0.00244390
1.41	1.08969450	1.08969593	0.00000143	0.00013128

Value of x	Actual value of f(x)=$x^{1/4}$	Calculated value of f(x)=$x^{1/4}$	Difference between the actual value of f(x)=$x^{1/4}$ and calculated value	Percentage error in the calculated value
1.42	1.09162140	1.09166396	0.00004256	0.00389858
1.43	1.09353828	1.09355807	0.00001979	0.00180961
1.44	1.09544504	1.09546888	0.00002384	0.00217645
1.45	1.09734190	1.09735668	0.00001478	0.00134707
1.46	1.09922898	1.09924090	0.00001192	0.00108448
1.47	1.10110641	1.10112488	0.00001848	0.00167808
1.48	1.10297430	1.10299039	0.00001609	0.00145908
1.49	1.10483277	1.10485935	0.00002658	0.00240613
1.50	1.10668182	1.10671628	0.00003445	0.00311304
1.51	1.10852170	1.10854471	0.00002301	0.00207550
1.52	1.11035252	1.11034787	0.00000465	0.00041871
1.53	1.11217427	1.11218143	0.00000715	0.00064311
1.54	1.11398709	1.11400723	0.00002015	0.00180849
1.55	1.11579108	1.11582327	0.00003219	0.00288464
1.56	1.11758637	1.11761475	0.00002837	0.00253867
1.57	1.11937308	1.11939394	0.00002086	0.00186369
1.58	1.12115133	1.12118006	0.00002873	0.00256249
1.59	1.12292111	1.12291873	0.00000238	0.00021232
1.60	1.12468255	1.12472343	0.00004089	0.00363558
1.61	1.12643576	1.12647307	0.00003731	0.00331244
1.62	1.12818086	1.12821376	0.00003290	0.00291636
1.63	1.12991786	1.12994659	0.00002873	0.00254261
1.64	1.13164687	1.13165331	0.00000644	0.00056884
1.65	1.13336802	1.13338792	0.00001991	0.00175653
1.66	1.13508129	1.13511777	0.00003648	0.00321369
1.67	1.13678694	1.13680446	0.00001752	0.00154152
1.68	1.13848495	1.13853335	0.00004840	0.00425117
1.69	1.14017534	1.14017320	0.00000215	0.00018820
1.70	1.14185822	1.14185226	0.00000596	0.00052200
1.71	1.14353371	1.14360309	0.00006938	0.00606714
1.72	1.14520192	1.14524674	0.00004482	0.00391396

Value of x	Actual value of $f(x)=x^{1/4}$	Calculated value of $f(x)=x^{1/4}$	Difference between the actual value of $f(x)=x^{1/4}$ and calculated value	Percentage error in the calculated value
1.73	1.14686286	1.14689004	0.00002718	0.00236992
1.74	1.14851654	1.14858067	0.00006413	0.00558412
1.75	1.15016317	1.15021837	0.00005519	0.00479879
1.76	1.15180278	1.15182233	0.00001955	0.00169737
1.77	1.15343535	1.15348005	0.00004470	0.00387568
1.78	1.15506113	1.15511131	0.00005019	0.00434497
1.79	1.15667999	1.15670455	0.00002456	0.00212307
1.80	1.15829206	1.15837097	0.00007892	0.00681318
1.81	1.15989745	1.15989673	0.00000072	0.00006167
1.82	1.16149616	1.16144180	0.00005436	0.00468012
1.83	1.16308844	1.16314781	0.00005937	0.00510419
1.84	1.16467404	1.16472816	0.00005412	0.00464688
1.85	1.16625333	1.16631365	0.00006032	0.00517211
1.86	1.16782618	1.16788673	0.00006056	0.00518556
1.87	1.16939259	1.16943014	0.00003755	0.00321115
1.88	1.17095292	1.17106640	0.00011349	0.00969187
1.89	1.17250693	1.17252243	0.00001550	0.00132172
1.90	1.17405474	1.17409778	0.00004303	0.00366546
1.91	1.17559648	1.17563164	0.00003517	0.00299140
1.92	1.17713225	1.17714489	0.00001264	0.00107347
1.93	1.17866194	1.17866170	0.00000024	0.00002023
1.94	1.18018579	1.18015420	0.00003159	0.00267674
1.95	1.18170369	1.18181765	0.00011396	0.00964405
1.96	1.18321586	1.18318939	0.00002646	0.00223666
1.97	1.18472219	1.18473506	0.00001287	0.00108672
1.98	1.18622279	1.18630683	0.00008404	0.00708489
1.99	1.18771768	1.18785548	0.00013781	0.01160258
2.00	1.18920696	1.18922150	0.00001454	0.00122296
Average percentage error				0.00240526

CONCLUSION

Average percentage error in the interpolation of functions $x^{1/2}$, $x^{1/3}$ and $x^{1/4}$ using Newton's divided difference formulas are 0.00234432%, 0.00222263% and 0.00240526% respectively. It means $x^{1/3}$ is better interpolated as compared to $x^{1/2}$ and $x^{1/4}$. The order of accuracy of these functions is as follows-

$$x^{1/3} > x^{1/2} > x^{1/4}$$

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